What's the Question? Level 3, Sample 1

A

Exemplar Task

Jasmine and Luciano discovered some ancient mathematical problems where only the products could be seen. They wondered if it was possible to use the clues they had been given to figure out the original questions. For each of the following questions use base-ten blocks.

1. Find the numbers which, when multiplied, give the products shown.
   Use the blank space to show your work and to show any other combinations that would give you the same product.
   [Note: Do not put more than one digit inside a 'box']

8 . 2	4 . 1	.
x 2	x 4	x
1 6 . 4	1 6 . 4	1 6 . 4

2. Explain what you did to find your solutions.

8 → . = 8.2
x 2
16.4

4 → . = 4.1
x 4
16.4

I found my solutions by knowing that when 8.2 is multiplied by 2 you will get 16.4, and when 4.1 is multiplied by 4 you will get 16.4 also.

B

3. Here is another problem Luciano and Jasmine found.
   Use this information to find the numbers which, when multiplied, give the products shown.
   Use the blank space to show your work and to show any other combinations that would give you the same product.

7 . 5	1 . 5	2 . 5
x 1	x 5	x 3
7 . 5	7 . 5	7 . 5

. = 1.5
x 5
7.5

I know this because I divided 7.5 by 5 and got 1.5.

. = 2.5
x 3
7.5

I know this I divided 7.5 by 3 also, and got 2.5.

C

4. Do you think some products can only be found in **one way**? Give reasons to justify your thinking.

(You don't have to use all of the boxes in the product)

3 . 1	1 . 3	2 . 1
x 1	x 1	x 1
3 . 1	1 . 3	2 . 1
I think that 3.1 can only be found in one way because both numbers are odd numbers.	I think that 13 can only be found in one way because it can only be divided by 1.	I think 2.1 can only be found in 1 way because 2.1 can only be divided by 1. if you divide by 2 you will get: 1.0

$$\begin{array}{r} 1.0 \\ 2\overline{)2.1} \\ -2 \\ \hline 01 \\ -0 \\ \hline R.1 \end{array}$$

D

5. Is it possible to find a product that can only be found in **two ways**? Investigate showing all your work in the space below. (You don't have to use all of the boxes in the product)

8 . 0	4 . 0
x 2	x 4
16 . 0	16 . 0

this can only be found in 2 ways, because 16.0 can only be divided by 2 numbers = 8 and 4.

E

6. Jasmine and Luciano's classmates began to wonder if they could find a product that could be written in **four different ways** using the template shown below.

a) Use the template below to investigate the question.
[Note: Do not put more than one digit inside a 'box']

3 . 0	4 . 5	9 . 0	1 . 5
x 3	x 2	x 1	x 6
9.0	9.0	9.0	9.0

b) Explain how you found the answer to this question.

I found the answer to this question by knowing that 3 x 3 = 9 and 4.5 x 2 = 9 and ofcourse 9 x 1 = 9, but I knew the last one by knowing if 1.5 x 5 = 7.5 than 1.5 x 6 = 9.

F

7.a) Draw as many rectangles as you can with a perimeter of 12.8 cm.
Draw the rectangle such that one of the dimensions is always a whole number and the other dimension is not shorter than 1 unit.

12.8 = a) 5cm, 1.4cm, 1.4cm, 5cm b) 4cm, 2.4cm, 2.4cm, 4cm c) 3cm, 3.4cm, 3.4cm, 3cm

d) 2cm, 4.4cm, 4.4cm, 2cm e) 1cm, 5.4cm, 5.4cm, 1cm

b) Find the areas of the rectangles you have drawn.
What are some of the things you notice?

a) A = l x w
= 5 x 1.4
= 1.4
x 5
A = 7.0

b) A = l x w
= 4 x 2.4
= 2.4
x 4
A = 9.6

c) A = l x w
= 3 x 3.4
= 3.4
x 3
A = 10.2

d) A = l x w
= 2 x 4.4
= 4.4
x 2
A = 8.8

e) A = l x w
= 1 x 5.4
= 5.4
x 1
A = 5.4

I notice a pattern
9.6 − 7.0 = 2.6cm difference 2cm
10.2 − 9.6 = 0.6
8.8 − 5.4 = 3.4
10.2 − 8.8 = 1.4 difference 2cm

next page

G

The pattern that I notice is: between rectangles A, b, c, the difference is 2cm, and between c, d, e. the difference is also 2cm. So for both differences, it is 2cm.

Teacher's Notes

Problem Solving

– The student selects and applies an appropriate problem-solving strategy that leads to a generally complete and accurate solution (e.g., in question 4, uses his or her knowledge of number properties to arrive at solutions; in question 7, uses a pattern to draw all of the possible rectangles).

Understanding of Concepts

– The student demonstrates a general understanding of processes and solutions involving place value and decimals by providing appropriate and complete explanations and illustrations (e.g., in question 3, "I know this because I divided 7.5 by 5 and got 1.5" and provides appropriate base-ten block illustrations).

Application of Mathematical Procedures

– The student uses computations and mathematical procedures that include few errors and/or omissions (e.g., in question 4, two of the three solutions provided are accurate; in question 6, there are no computational errors; in question 5, 16.0 is an incorrect solution because it has more than two sets of factors).

Communication of Required Knowledge

– The student uses mathematical language and notation clearly to explain and justify responses to reverse-processing problems involving decimals (e.g., in question 2, uses base-ten blocks, numbers, and a written explanation; in question 3, "I know this because I divided 7.5 by 5 and got 1.5"; in question 6, states a variety of combinations, including "I knew the last one by knowing if 1.5 x 5 = 7.5 than 1.5 x 6 = 9").

Comments/Next Steps

– The student should further explore the use of base-ten blocks for solving a variety of problems involving decimals.
– The student should explore the relationship between the dimensions (factors) and the area (product) of rectangles.

What's the Question? Level 3, Sample 2

A

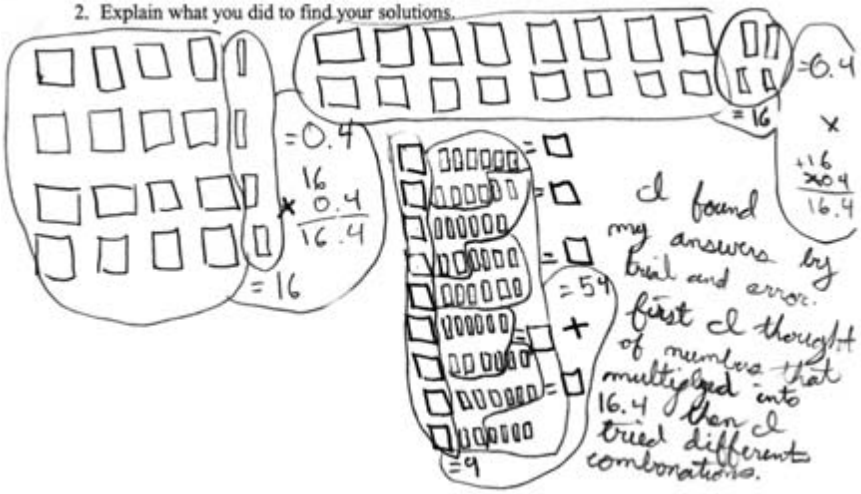
Exemplar Task

Jasmine and Luciano discovered some ancient mathematical problems where only the products could be seen. They wondered if it was possible to use the clues they had been given to figure out the original questions. For each of the following questions use base-ten blocks.

1. Find the numbers which, when multiplied, give the products shown.
   Use the blank space to show your work and to show any other combinations that would give you the same product.
   [Note: Do not put more than one digit inside a 'box']

4 . 1	8 . 2	1 . 6
x 4	x 2	x 9
16 . 4	16 . 4	16 . 4

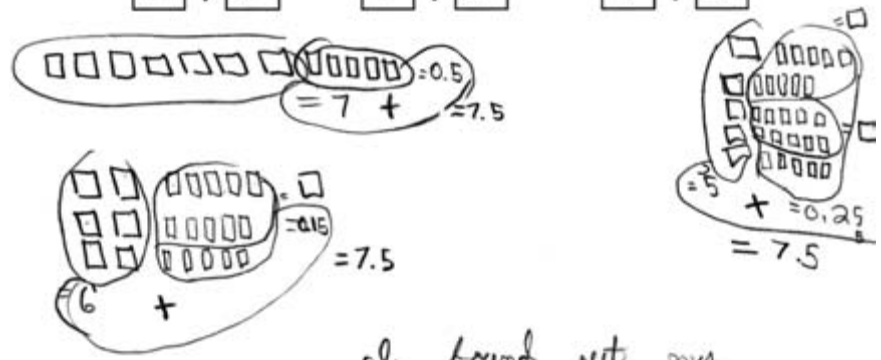
2. Explain what you did to find your solutions.



B

3. Here is another problem Luciano and Jasmine found.
   Use this information to find the numbers which, when multiplied, give the products shown.
   Use the blank space to show your work and to show any other combinations that would give you the same product.

7 . 5	1 . 5	2 . 5
x 1	x 5	x 3
7 . 5	7 . 5	7 . 5



I found out my answers by figuring out what numbers multiply to 7 or are very close to 7. If it didn't answer to 7 find a decimal that would add to it and make it 7. And the decimal would make the whole 7. be 5 or a number

C

4. Do you think some products can only be found in **one way**? Give reasons to justify your thinking.

(You don't have to use all of the boxes in the product)

1.0	1.1	2.0
x 1	x 1	x 2
1.0	1.1	2.0

I think some products can be found only one way because you can only times it by 1. That would be prime numbers you would have to use though. The examples I have shown can only be found 1 way.

D

5. Is it possible to find a product that can only be found in **two ways**? Investigate showing all your work in the space below. (You don't have to use all of the boxes in the product)

1.2	0.3
x 1	x 4
1.2	1.2

2.4 × 1 = 2.4 ✓ 1.4 × 2 = 2.8 ✗ 1.7 × 2 = 3.4 ✗ 1.2 × 2 = 2.4 ✓ 8.6 4 2.4 ✓ 1.2 × 1 = 1.2 ✓ 0.3 × 4 = 1.2 2.4 don't work! 1.1 × 3 = 3.3 ✗

E

6. Jasmine and Luciano's classmates began to wonder if they could find a product that could be written in **four different ways** using the template shown below.

a) Use the template below to investigate the question.
[Note: Do not put more than one digit inside a 'box']

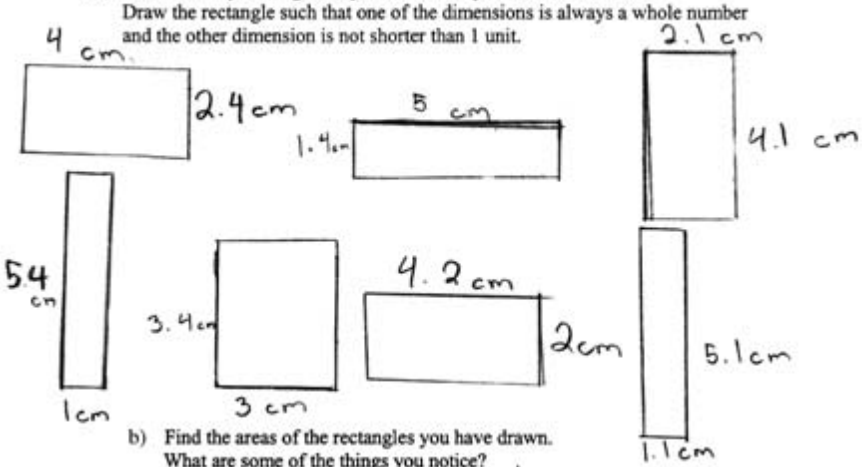
8.4	4.2	2.1	2.8
x 1	x 2	x 4	x 3
8.4	8.4	8.4	8.4

b) Explain how you found the answer to this question.

I found the answer to this question by trial and error. I tried different combinations to see what ones worked. I started with 6.2 and it didn't work so I tried 6.4. That didn't work so I tried 8.4, and it worked!

F

7.a) Draw as many rectangles as you can with a perimeter of 12.8 cm.
Draw the rectangle such that one of the dimensions is always a whole number and the other dimension is not shorter than 1 unit.



b) Find the areas of the rectangles you have drawn.
What are some of the things you notice?

4.2 × 2 = 8.4 cm²
5.2 × 1 = 5.2 cm²
1.4 × 5 = 7.0 cm²
2.1 × 4.1 = 6.20 = 6.61 cm²
3.4 × 3 = 10.2 cm²
4.2 × 2 = 8.4

5.1 × 1.1 = 5.1 + 1.1 = 5.50 = 5.61

8.8 cm², 5.2 cm², 7.0 cm², 6.61 cm², 10.2 cm, 8.4 cm², and 5.61 cm²

I notice that they are all different, close in range, and small amounts

Teacher's Notes

Problem Solving

– The student selects and applies an appropriate problem-solving strategy that leads to a generally complete and accurate solution (e.g., in question 1, uses base-ten blocks and a knowledge of multiplication and division to arrive at three sets of factors for the product, one of which is inaccurate).

Understanding of Concepts

– The student demonstrates a general understanding of processes and solutions involving place value and decimals by providing appropriate and complete explanations and illustrations (e.g., in question 3, uses illustrations of base-ten blocks to support his or her conclusion: "I found out my answers by figuring out what numbers multiply to 7 or are very close to 7").

Application of Mathematical Procedures

– The student uses computations and mathematical procedures that include few errors and/or omissions (e.g., in question 4, two of the three solutions are accurate, with a minor computational error in the third: "2.0 x 2 = 2.0").

Communication of Required Knowledge

– The student uses mathematical language and notation clearly to explain and justify responses to reverse-processing problems involving decimals (e.g., in question 2, uses base-ten blocks, numbers, and a written statement: "I thought of numbers that multiplyed into 16.4 then I tried different combonations"; in question 4, mentions prime numbers and their significance to the solution).

Comments/Next Steps

– The student needs to justify his or her responses by providing more than one example or by supplying multiple reasons for his or her responses.
– The student should further explore the use of base-ten blocks to find products with a given number of factors.
– The student should explore the relationship between the dimensions (factors) and the area (product) of a rectangle.

What's the Question? Level 4, Sample 1

A

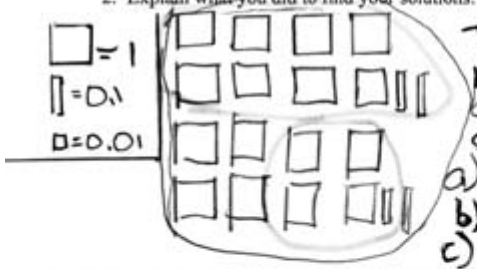
Exemplar Task

Jasmine and Luciano discovered some ancient mathematical problems where only the products could be seen. They wondered if it was possible to use the clues they had been given to figure out the original questions. For each of the following questions use base-ten blocks.

1. Find the numbers which, when multiplied, give the products shown.
   Use the blank space to show your work and to show any other combinations that would give you the same product.
   [Note: Do not put more than one digit inside a 'box']

4 . 1	16 . 4	8 . 2
x 4	x 1	x 2
16 . 4	16 . 4	16 . 4

2. Explain what you did to find your solutions.



This is 16,4 in base ten blocks. I grouped them in three groups, for instance
a) 1 group of 16.4
b) 2 groups of 8.2
c) 4 groups of 4.1

Since 2×8=16 ~~2×0.2=0.4~~ 16/.4 factors of 16
1×16=16 4×4=16
2×8=16
factors of 0.4
1×0.4=0.4 2×0.2=0.4

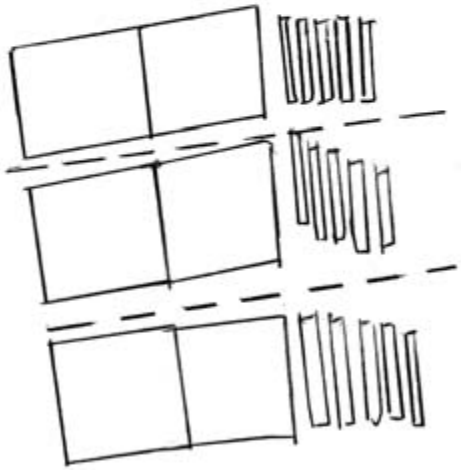
2 can be a multiple of 16.4
Since 4×4=16 ~~4×1=4~~ 4 can be a multiple of 16.4.

The Ontario Curriculum Exemplars Project, Grades 1-8
Mathematics, Grade 6
Number Sense and Numeration

B

3. Here is another problem Luciano and Jasmine found.
   Use this information to find the numbers which, when multiplied, give the products shown.
   Use the blank space to show your work and to show any other combinations that would give you the same product.

1 . 5	7 . 5	2 . 5
x 5	x 1	x 3
7 . 5	7 . 5	7 . 5

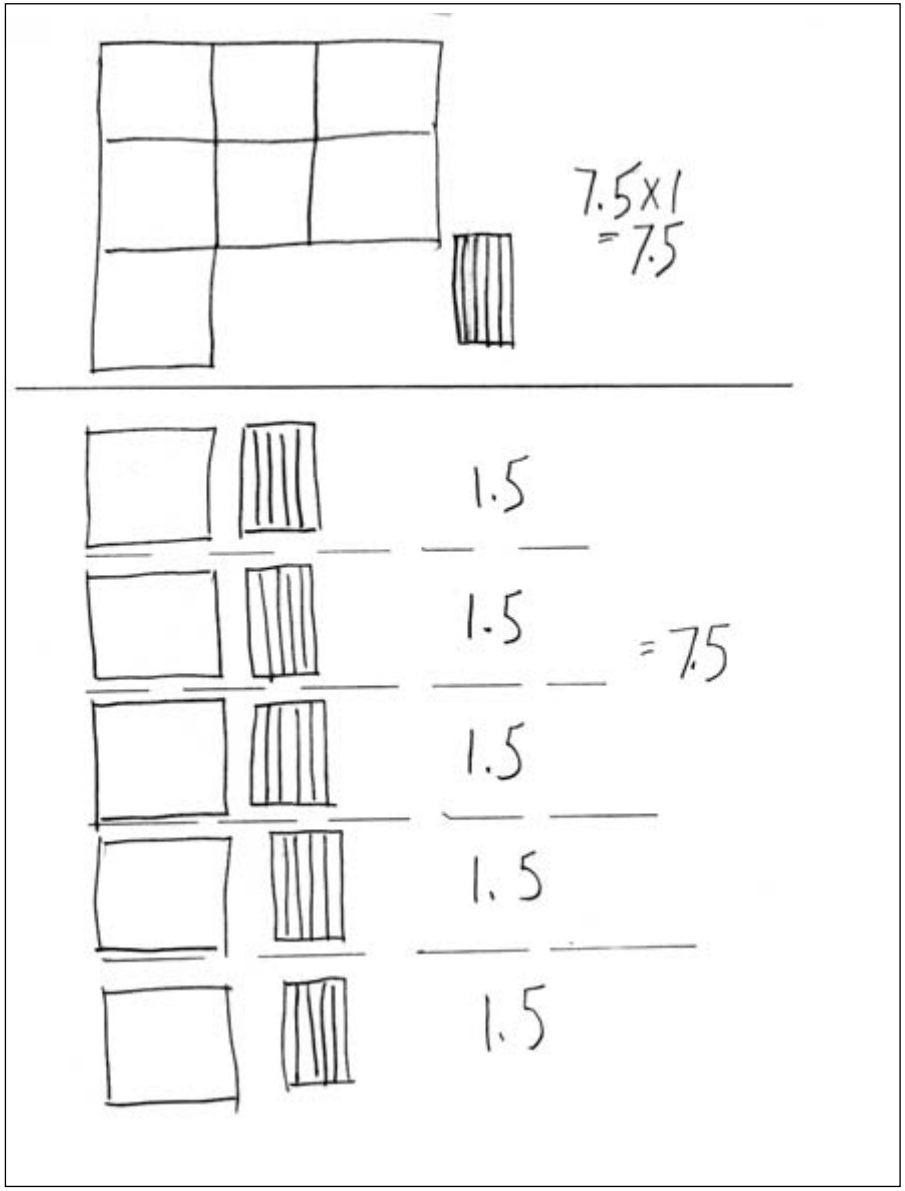


I grouped these blocks into three groups which is
a) 3 groups of 2.5
b) 1 group of 7.5
c) 5 groups of 1.5

If you take away the decimal, instead of 7.5 you will have 75
75 can be made multiplying
1×75
3×25
5×15

I know that that there are 8 100-base blocks and 5 ten base blocks in a)

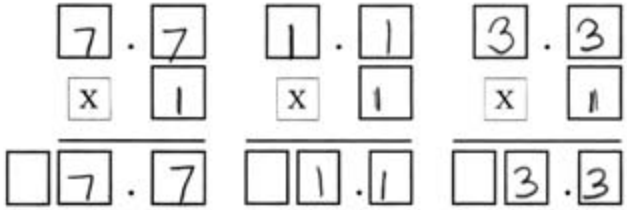
C



D

4. Do you think some products can only be found in **one way**? Give reasons to justify your thinking.

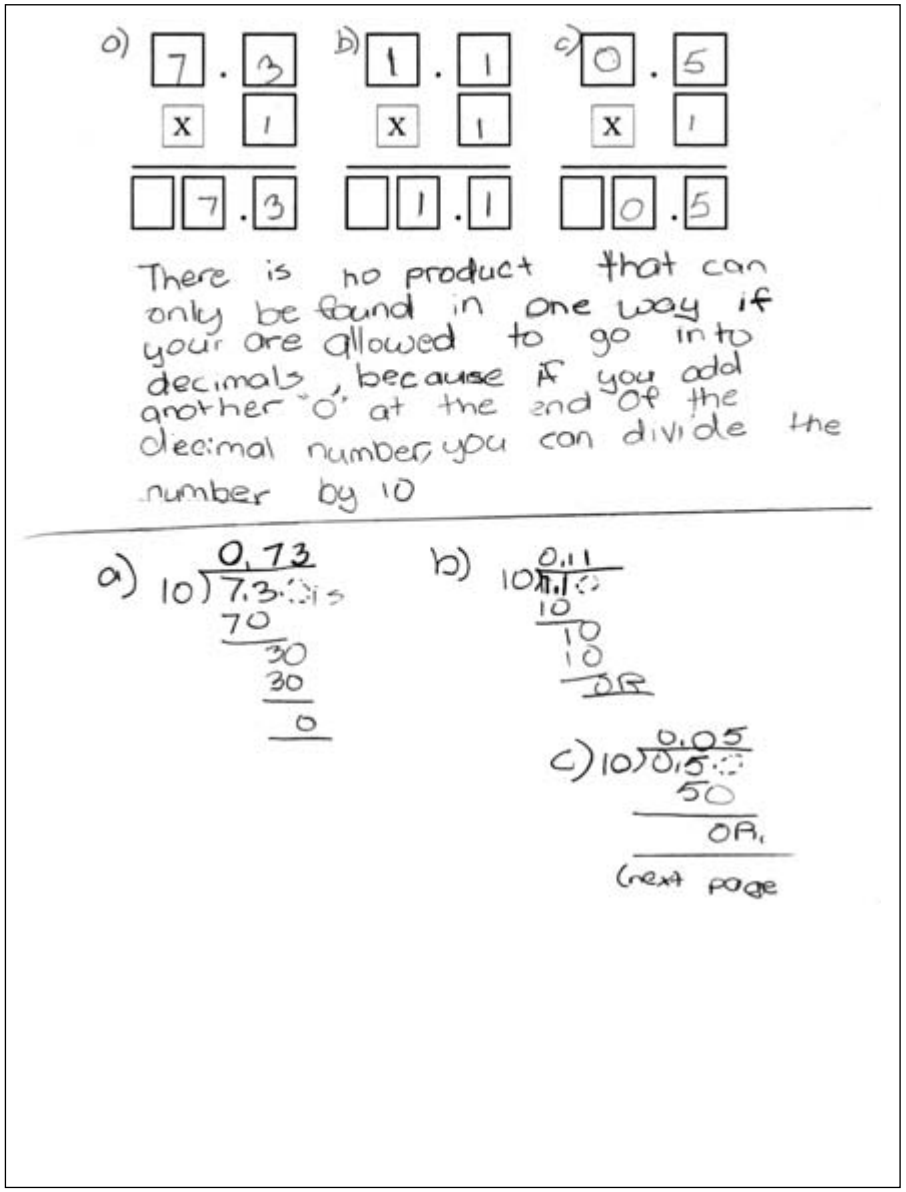
(You don't have to use all of the boxes in the product)



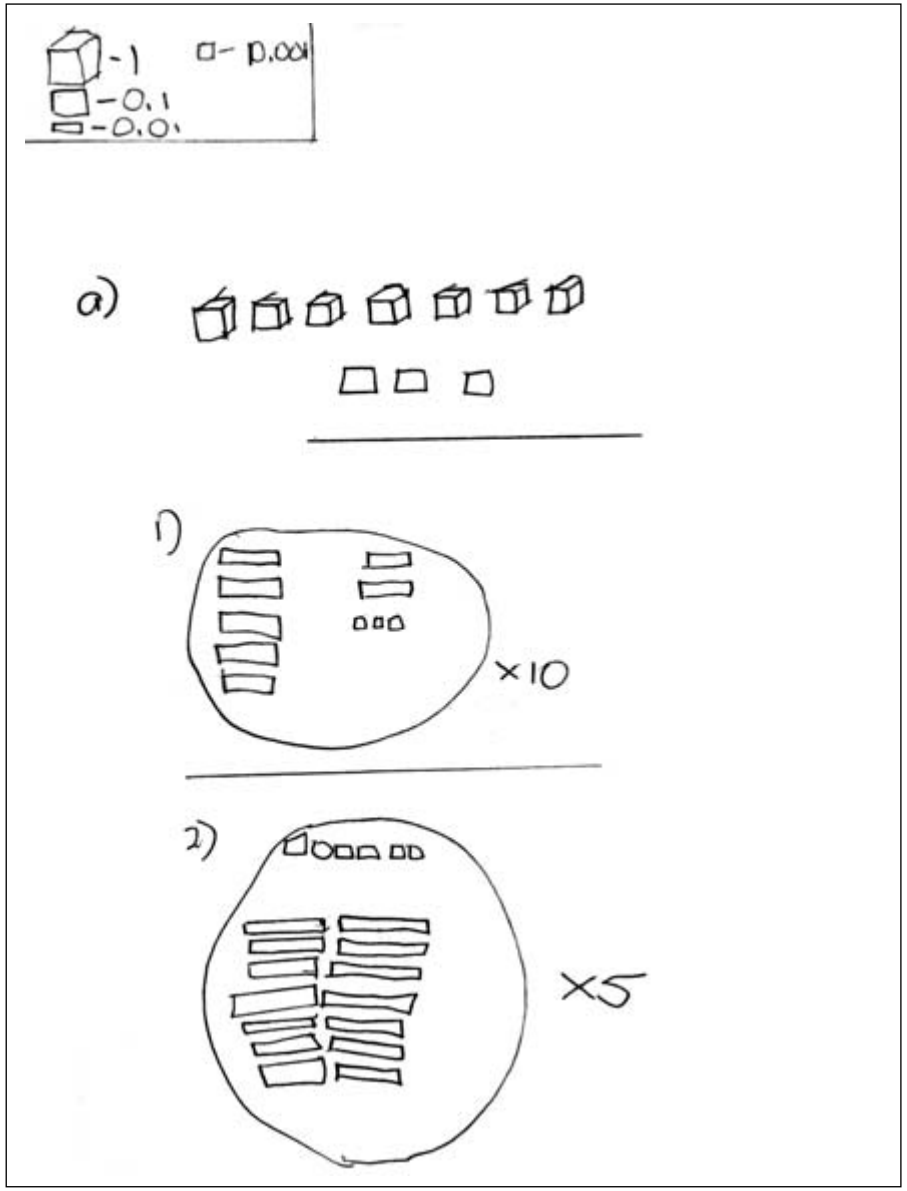
The first number is prime if you take away the decimal number

You can only group these one way

E


a) 7.3 x 1 = 7.3
b) 1.1 x 1 = 1.1
c) 0.5 x 1 = 0.5
There is no product that can only be found in one way if your are allowed to go into decimals, because if you add another "0" at the end of the decimal number you can divide the number by 10
a) 10)7.3 = 0.73
70
30
30
0
b) 10)1.1 = 0.11
10
10
0R
c) 10)0.5 = 0.05
50
0R.
(next page

F


-1
-0.1
-0.01
-0.001
a)
1)
×10
2)
×5

G

5. Is it possible to find a product that can only be found in **two ways**?
Investigate showing all your work in the space below.
(You don't have to use all of the boxes in the product)

a)

7	.	5	2	.	5
x		2	x		6
15	.	0	15	.	0

It is impossible to find a product which can only be found in 2 ways because every multiple can be divided by 2 so you must add another 2 to the other multiple for example

7.5 - ÷2 - 3.75
x 2 - +2 - x 4
15.0 - same - 15.00

25
x 6
15.0

This procedure always works. no matter whether you are using prime numbers or composite numbers it works.

H

6. Jasmine and Luciano's classmates began to wonder if they could find a product that could be written in **four different ways** using the template shown below.

a) Use the template below to investigate the question.
[Note: Do not put more than one digit inside a 'box']

1)	2)	3)	4)
3 . 0	2 . 5	5 . 0	7 . 5
x 5	x 6	x 3	x 2
15.0	15.0	15.0	15.0

b) Explain how you found the answer to this question.

Well I used the process of counting money, I removed all the decimals first. Then I thought of using bills

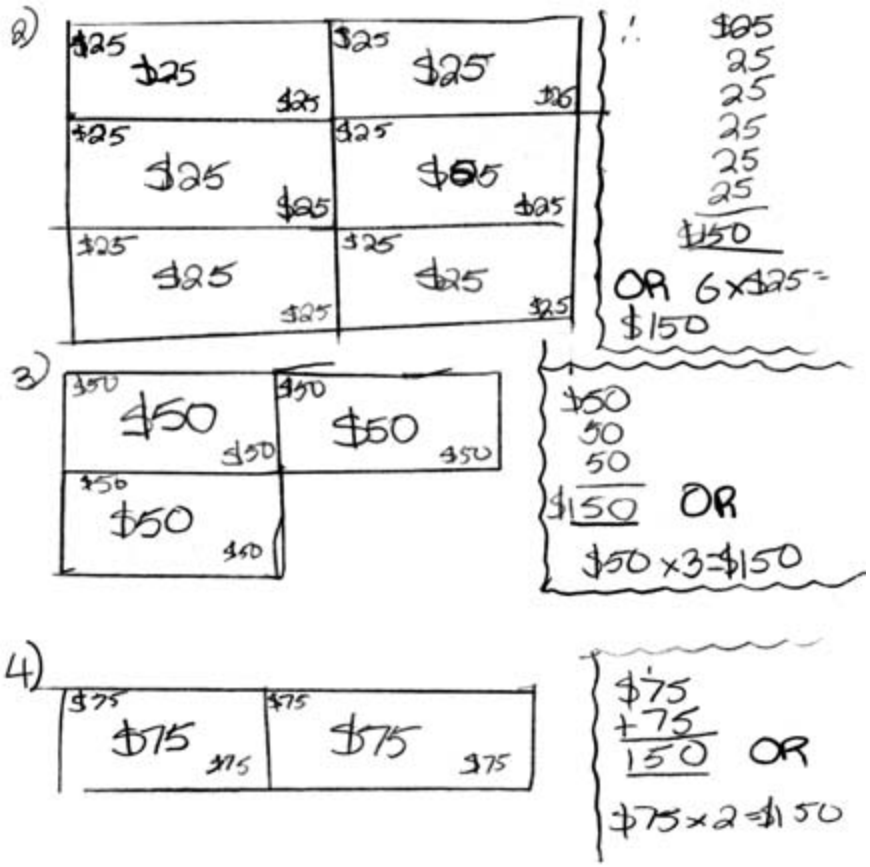
1)

$30 $30 $30	$30 $30 $30	$30 $30 $30
$30 $30 $30	$30 $30 $30	

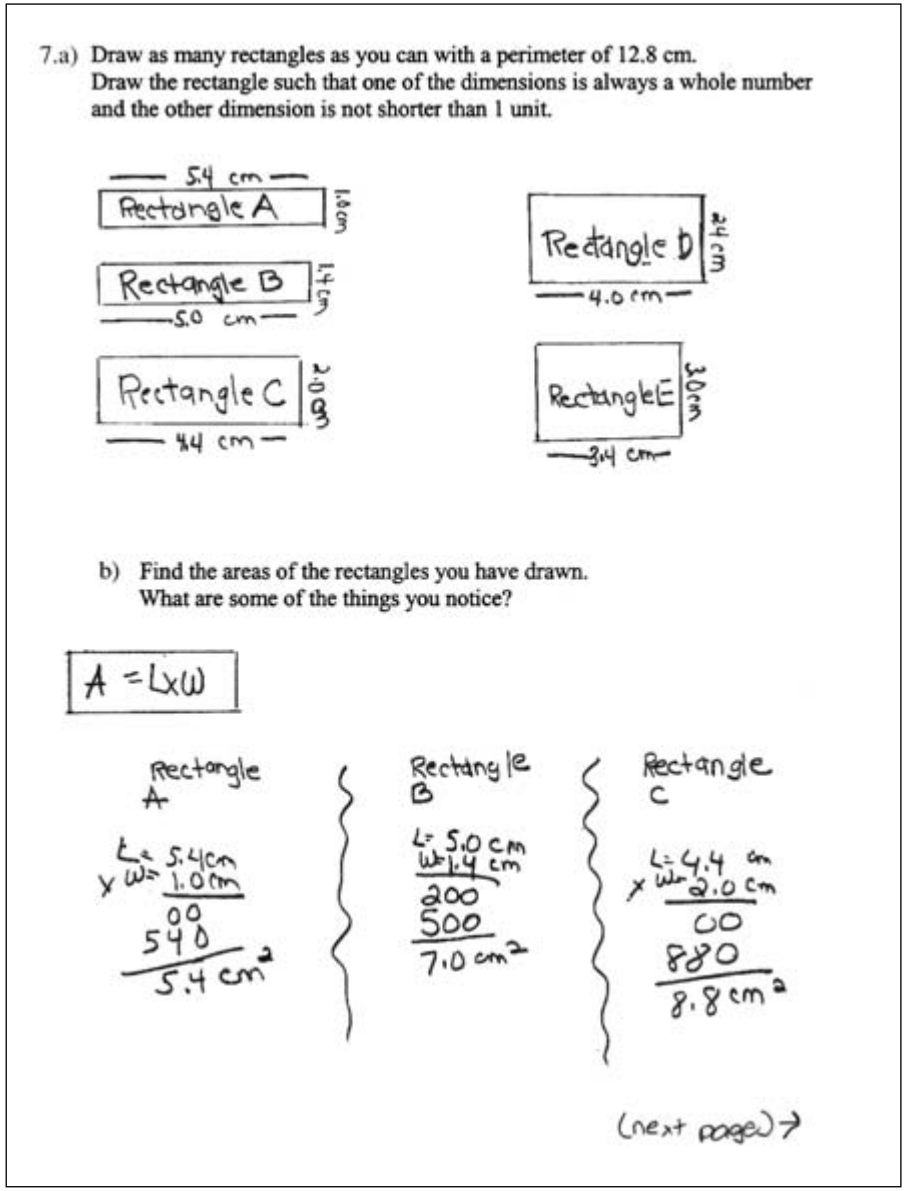
$30
30
30
30
30
$150 OR

$30
x 5
$150

I


2)
$25 $25 $25
$25 $25 $25
$25 $25 $25
$25 $25 $25
$25 $25 $25
$25 $25 $25
∴ $25 25 25 25 25 25 $150
OR 6×$25 = $150
3)
$50 $50 $50
$50 $50 $50
$50 $50 $50
$50 50 50 $150 OR
$50 × 3 = $150
4)
$75 $75 $75
$75 $75 $75
$75 + 75 150 OR
$75 × 2 = $150

J


7.a) Draw as many rectangles as you can with a perimeter of 12.8 cm.
Draw the rectangle such that one of the dimensions is always a whole number and the other dimension is not shorter than 1 unit.
5.4 cm
Rectangle A
1.0 cm
Rectangle B
1.4 cm
5.0 cm
Rectangle C
2.0 cm
4.4 cm
Rectangle D
2.4 cm
4.0 cm
Rectangle E
3.0 cm
3.4 cm
b) Find the areas of the rectangles you have drawn.
What are some of the things you notice?
A = L×W
Rectangle A
L = 5.4 cm
× W = 1.0 cm
00
540
5.4 cm²
Rectangle B
L = 5.0 cm
W = 1.4 cm
200
500
7.0 cm²
Rectangle C
L = 4.4 cm
× W = 2.0 cm
00
880
8.8 cm²
(next page) →

K

Rectangle E

L = 3.4
x W = 3.0
00
1020
10.2 cm²

Rectangle D

L = 4.0
x W = 2.4
160
80
9.6 cm²

I noticed that:

1) • all have 1 whole number
2) • the decimals on the top go down
- length = 5.4, 5.0, 4.4, 4.0, 3.4
3) • the decimals on the bottom go up –
width = 1.0, 1.4, 2.0, 2.4, 3.0
4) • the length and the width have either
n.4 or n.0
5) • the areas go up by .6 cm²

Teacher's Notes

Problem Solving

– The student selects and applies an appropriate problem-solving strategy that leads to a thorough and accurate solution (e.g., in question 6, uses the concept of money to find four sets of factors – "I removed all the decimals first. Then I thought of using bills" – supported by clearly labelled illustrations).

Understanding of Concepts

– The student demonstrates a thorough understanding of processes and solutions involving place value and decimals by providing appropriate and detailed explanations and illustrations (e.g., in questions 2 and 3, uses broken lines, colour coding, and circling of base-ten blocks to demonstrate his or her understanding of grouping).

Application of Mathematical Procedures

– The student uses computations and mathematical procedures that include few, if any, minor errors and/or omissions (e.g., in questions 1, 4, 5, and 6, provides accurate answers; in question 7, provides all of the possible rectangles).

Communication of Required Knowledge

– The student uses mathematical language and notation clearly and precisely to explain and justify responses to reverse-processing problems involving decimals (e.g., in question 2, explains the process used to find three sets of factors for a given product: "Since 2 x 8 = 16 & 2 x 0.2 = 0.4 ∴ 2 can be a multiple of 16.4").

Comments/Next Steps

– The student has a very good understanding of prime numbers, factors, and multiples. Additional rich investigations could be assigned to this student.

A

Exemplar Task

Jasmine and Luciano discovered some ancient mathematical problems where only the products could be seen. They wondered if it was possible to use the clues they had been given to figure out the original questions. For each of the following questions use base-ten blocks.

1. Find the numbers which, when multiplied, give the products shown.
   Use the blank space to show your work and to show any other combinations that would give you the same product.
   [Note: Do not put more than one digit inside a 'box']

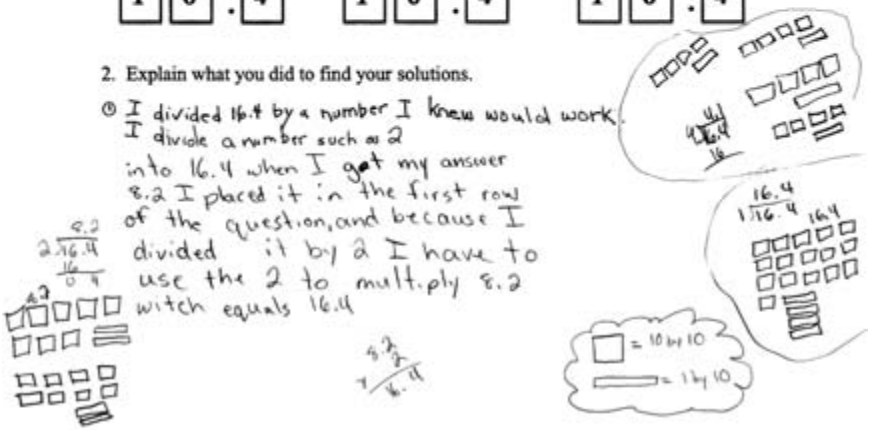
① 8 . 2 × 2 = 1 6 . 4

② 4 . 1 × 4 = 1 6 . 4

1 6 . 4 × 1 = 1 6 . 4

2. Explain what you did to find your solutions.

① I divided 16.4 by a number I knew would work. I divide a number such as 2 into 16.4 when I got my answer 8.2 I placed it in the first row of the question, and because I divided it by 2 I have to use the 2 to multiply 8.2 witch equals 16.4



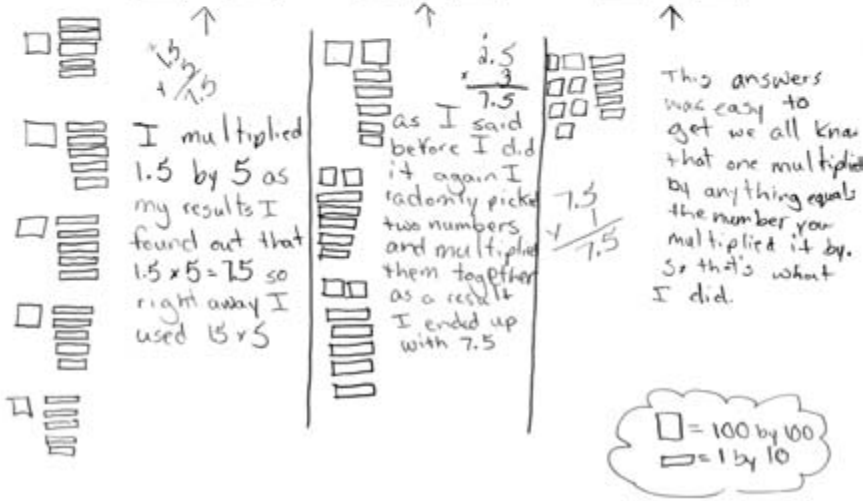
B

3. Here is another problem Luciano and Jasmine found.
   Use this information to find the numbers which, when multiplied, give the products shown.
   Use the blank space to show your work and to show any other combinations that would give you the same product.

1 . 5 × 5 = 7 . 5

2 . 5 × 3 = 7 . 5

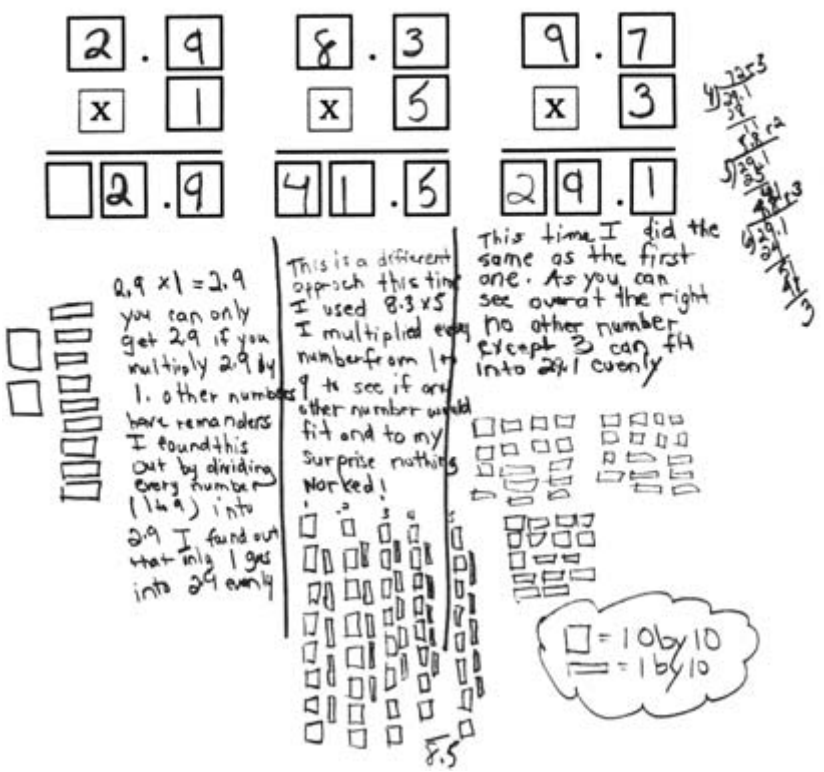
7 . 5 × 1 = 7 . 5



C

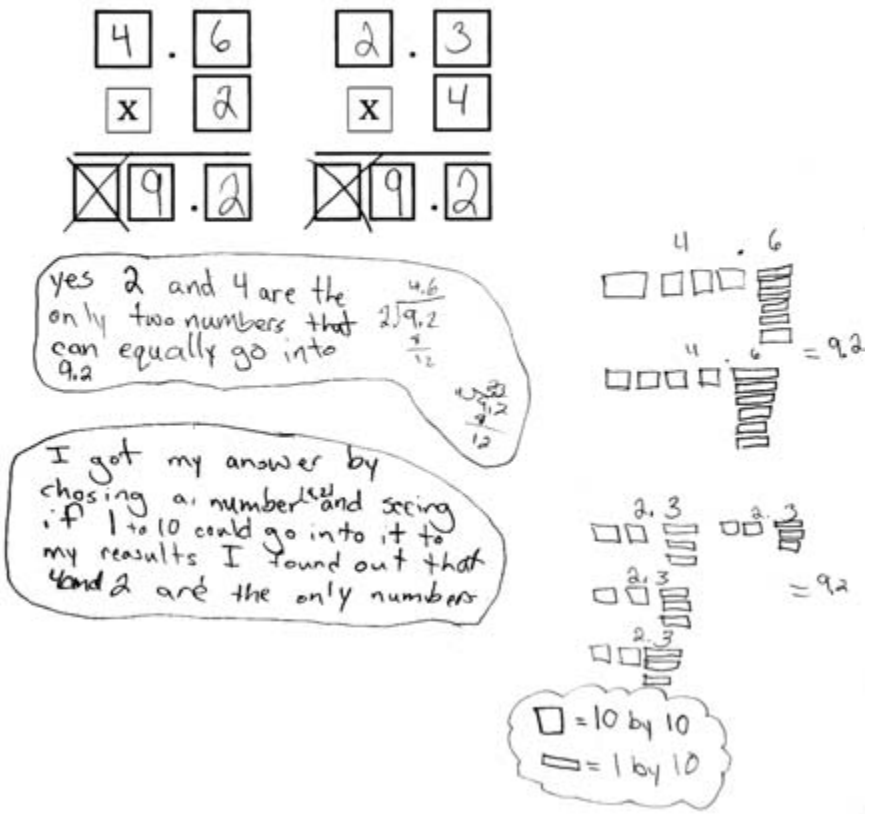
4. Do you think some products can only be found in **one way**? Give reasons to justify your thinking.

(You don't have to use all of the boxes in the product)



D

5. Is it possible to find a product that can only be found in **two ways**? Investigate showing all your work in the space below. (You don't have to use all of the boxes in the product)



E

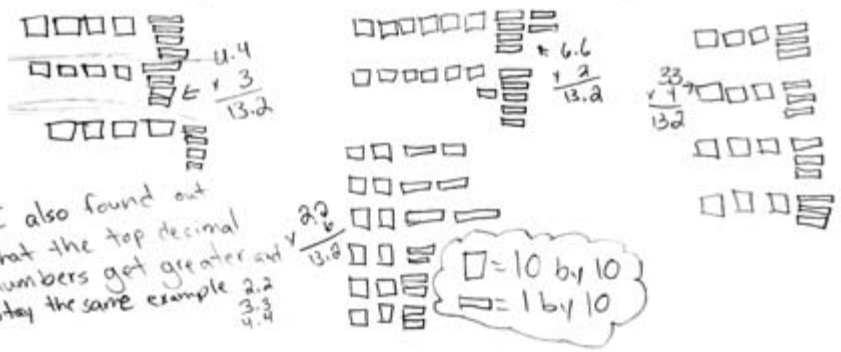
6. Jasmine and Luciano's classmates began to wonder if they could find a product that could be written in **four different ways** using the template shown below.

a) Use the template below to investigate the question.
[Note: Do not put more than one digit inside a 'box']

2.2	3.3	4.4	6.6
x 6	x 4	x 3	x 2
13.2	13.2	13.2	13.2

b) Explain how you found the answer to this question.

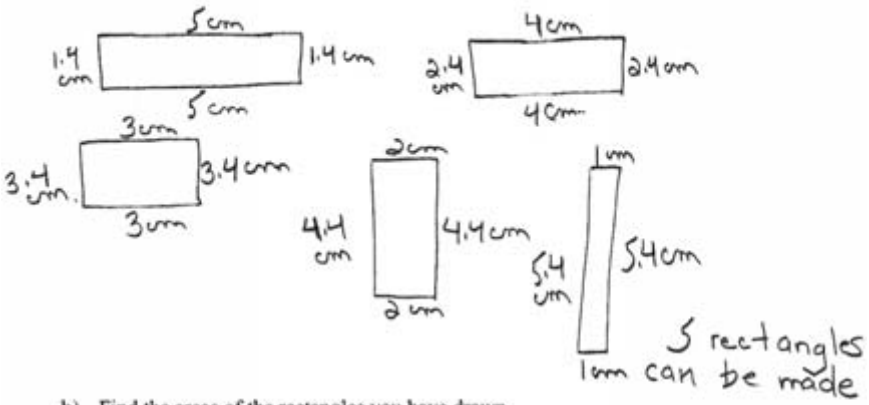
I found my answer by multiplying diffrent numbers by different numbers like 4.4 x 3 = 13.2 then I mutiplied other numbers together to see if it would equal the same.



I also found out that the top decimal numbers get greater and stay the same example 2.2 3.3 4.4

F

7.a) Draw as many rectangles as you can with a perimeter of 12.8 cm.
Draw the rectangle such that one of the dimensions is always a whole number and the other dimension is not shorter than 1 unit.



5 rectangles can be made

b) Find the areas of the rectangles you have drawn.
What are some of the things you notice?

2.4 x 4 = 9.6
1.4 x 5 = 7.0
4.4 x 2 = 8.8
3.4 x 3 = 10.2

I notice that the first number(s) of each area has a pattern.

7.0, 8.8, 9.6, 10.2

One number gets bigger
1.4, 2.4, 3.4, 4.4, 5.4
and the other gets smaller
5, 4, 3, 2, 1

Teacher's Notes

Problem Solving

– The student selects and applies an appropriate problem-solving strategy that leads to a thorough and accurate solution (e.g., in question 4, uses a systematic problem-solving approach: "I found this out by dividing every number (1 to 9) into 2.9 …").

Understanding of Concepts

– The student demonstrates a thorough understanding of processes and solutions involving place value and decimals by providing appropriate and detailed explanations and illustrations (e.g., in question 3, shows multiple ways of solving the problem to find three sets of factors for 7.5; uses diagrams to support written statements).

Application of Mathematical Procedures

– The student uses computations and mathematical procedures that include few, if any, minor errors and/or omissions (e.g., in questions 1, 3, 4, and 6, provides accurate responses; in question 7, draws five rectangles, including the one with the largest area).

Communication of Required Knowledge

– The student uses mathematical language and notation clearly and precisely to explain and justify responses to reverse-processing problems involving decimals (e.g., in question 2, uses a clear statement and supporting diagrams; in question 5, "I got my answer by chosing a number (9.2) and seeing if 1 to 10 could go into it to my reasults I found out that 4 and 2 are the only numbers").

Comments/Next Steps

– The student should expand his or her responses to include all the processes and procedures used (e.g., regrouping).

– The student could further investigate the use of base-ten blocks for solving problems involving the multiplication and division of decimals.